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**MATHEMATICS  
HIGHER LEVEL  
PAPER 3 – SETS, RELATIONS AND GROUPS**

Thursday 15 May 2014 (afternoon)

1 hour

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INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematics HL and Further Mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is [60 marks].

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 12]

The binary operation  $\Delta$  is defined on the set  $S = \{1, 2, 3, 4, 5\}$  by the following Cayley table.

| $\Delta$ | 1 | 2 | 3 | 4 | 5 |
|----------|---|---|---|---|---|
| 1        | 1 | 1 | 2 | 3 | 4 |
| 2        | 1 | 2 | 1 | 2 | 3 |
| 3        | 2 | 1 | 3 | 1 | 2 |
| 4        | 3 | 2 | 1 | 4 | 1 |
| 5        | 4 | 3 | 2 | 1 | 5 |

- (a) State whether  $S$  is closed under the operation  $\Delta$  and justify your answer. [2]
- (b) State whether  $\Delta$  is commutative and justify your answer. [2]
- (c) State whether there is an identity element and justify your answer. [2]
- (d) Determine whether  $\Delta$  is associative and justify your answer. [3]
- (e) Find the solutions of the equation  $a\Delta b = 4\Delta b$ , for  $a \neq 4$ . [3]

## 2. [Maximum mark: 19]

Consider the set  $S$  defined by  $S = \{s \in \mathbb{Q} : 2s \in \mathbb{Z}\}$ .

You may assume that  $+$  (addition) and  $\times$  (multiplication) are associative binary operations on  $\mathbb{Q}$ .

- (a) (i) Write down the six smallest non-negative elements of  $S$ .
- (ii) Show that  $\{S, +\}$  is a group.
- (iii) Give a reason why  $\{S, \times\}$  is not a group. Justify your answer. [9]
- (b) The relation  $R$  is defined on  $S$  by  $s_1 R s_2$  if  $3s_1 + 5s_2 \in \mathbb{Z}$ .
- (i) Show that  $R$  is an equivalence relation.
- (ii) Determine the equivalence classes. [10]

3. [Maximum mark: 15]

Sets  $X$  and  $Y$  are defined by  $X = ]0, 1[$ ;  $Y = \{0, 1, 2, 3, 4, 5\}$ .

(a) (i) Sketch the set  $X \times Y$  in the Cartesian plane.

(ii) Sketch the set  $Y \times X$  in the Cartesian plane.

(iii) State  $(X \times Y) \cap (Y \times X)$ .

[5]

Consider the function  $f : X \times Y \rightarrow \mathbb{R}$  defined by  $f(x, y) = x + y$

and the function  $g : X \times Y \rightarrow \mathbb{R}$  defined by  $g(x, y) = xy$ .

(b) (i) Find the range of the function  $f$ .

(ii) Find the range of the function  $g$ .

(iii) Show that  $f$  is an injection.

(iv) Find  $f^{-1}(\pi)$ , expressing your answer in exact form.

(v) Find all solutions to  $g(x, y) = \frac{1}{2}$ .

[10]

4. [Maximum mark: 14]

Let  $f : G \rightarrow H$  be a homomorphism of finite groups.

(a) Prove that  $f(e_G) = e_H$ , where  $e_G$  is the identity element in  $G$  and  $e_H$  is the identity element in  $H$ .

[2]

(b) (i) Prove that the kernel of  $f$ ,  $K = \text{Ker}(f)$ , is closed under the group operation.

(ii) Deduce that  $K$  is a subgroup of  $G$ .

[6]

(c) (i) Prove that  $gkg^{-1} \in K$  for all  $g \in G$ ,  $k \in K$ .

(ii) Deduce that each left coset of  $K$  in  $G$  is also a right coset.

[6]